Set-up

In the theorem, an individual agent is faced with options called *lotteries*. Given some mutually exclusive outcomes, a lottery is a scenario where each outcome will happen with a given probability, all probabilities summing to one. For example, for two outcomes A and B,

$$L = 0.25A + 0.75B$$

denotes a scenario where $P(A)$ = 25% is the probability of A occurring and $P(B)$ = 75% (and exactly one of them will occur). More generally, for a lottery with many possible outcomes A_i, we write:

$$L = \sum p_i A_i,$$

with the sum of the p_i s equalling 1.

The outcomes in a lottery can themselves be lotteries between other outcomes, and the expanded expression is considered an equivalent lottery: $0.5(0.5A + 0.5B) + 0.5C = 0.25A + 0.25B + 0.50C$.

If lottery M is preferred over lottery L, we write $M \succ L$, or equivalently, $L \prec M$. If the agent is indifferent between L and M, we write the *indifference relation*[4] $L \sim M$. If M is either preferred over or viewed with indifference relative to L, we write $L \preceq M$.

The axioms

The four axioms of VNM-rationality are then *completeness*, *transitivity*, *continuity*, and *independence*.

Completeness assumes that an individual has well defined preferences:

> **Axiom 1 (Completeness)** For any lotteries L,M, exactly one of the following holds:
>
> $L \prec M$, $M \prec L$, or $L \sim M$

(either M is preferred, L is preferred, or the individual is indifferent[5]).

Transitivity assumes that preferences are consistent across any three options:

> **Axiom 2 (Transitivity)** If $L \prec M$ and $M \prec N$, then $L \prec N$, and similarly for $\sim$.

Continuity assumes that there is a "tipping point" between being *better than* and *worse than* a given middle option:

> **Axiom 3 (Continuity):** If $L \preceq M \preceq N$, then there exists a probability $p \in [0,1]$ such that
>
> $pL + (1-p)N \sim M$